**A**

**RESEARCH TITLE:** CURVE AND SURFACE MODELING WITH CONSTRAINED B-SPLINE WAVELETS

**PHASE & YEAR:** 1 & 2008

**START DATE:** 1.11.2008

**END DATE:** 31.10.2011

**EXTENSION PERIOD (DATE):** RMC LEVEL: 1,11,2011 – 30.4.2012

**PROJECT LEADER:** PROF MADYA MOHD NAJIB BIN UZ AWA

**I/C / PASSPORT NUMBER:** 550630035109

**PROJECT MEMBERS:**
1. PROF DR TANAWIN MOHD ALI
2. PROF MADYA MOHAMAD FAISAL ABUUL KARI

**RO:** MUHAMMAD ABBAS MUHAMMAD YAQUB

**RA:** 1) NUR ATIQAH BT ZULKIFLI
2) TENGku ROSEHAYUNNI ANIS BT TENGku ROSLI

**PROJECT ACHIEVEMENT**

<table>
<thead>
<tr>
<th>ACHIEVEMENT PERCENTAGE</th>
<th>0 - 50%</th>
<th>51 - 75%</th>
<th>76 - 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage (please state #%)</td>
<td>83%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**RESEARCH OUTPUT**

<table>
<thead>
<tr>
<th>Number of articles/ manuscripts/ books (Please attach the First Page of Publication)</th>
<th>Indexed Journal</th>
<th>Non-Indexed Journal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conference Proceeding (Please attach the First Page of Publication)</td>
<td>International</td>
<td>National</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Intellectual Property (Please specify) | |
|----------------------------------------|
E. PROBLEMS / CONSTRAINTS IF ANY (Masalah/ Kekangan sekiranya ada)

Triada

F. RECOMMENDATION (Cadangan/ Penambahbaikan)

Triada

G. RESEARCH ABSTRACT – Not More Than 200 Words (Abstrak Penyelidikan – Tidak Melebihi 200 bahasa perkataan)

Seperti dalam Borang Permohonan

Date: 30.6.2015

Project Leader’s Signature: [Signature]

H. COMMENTS, IF ANY / ENDORSEMENT BY RESEARCH MANAGEMENT CENTER (RMC)
(Komen, sekiranya ada/ Pengesahan oleh Pusat Pengurusan Penyelidikan)

Name: PROF. DR LEE KEAT TEONG
Nama: Pengarah
Pepat Pengurusan & Kreativiti Penyelidikan
Universiti Sains Malaysia

Signature: [Signature]

Date: 15/7/15

Tandatangan:
A class of quasi-quintic trigonometric Bézier curve with two shape parameters

Uzma Bashir\textsuperscript{a,}\textsuperscript{*}, Muhammad Abbas\textsuperscript{a}, Mohd Nain Hj Awang\textsuperscript{b}, Jamaludin Md. Ali\textsuperscript{a}

\textsuperscript{a} School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia
\textsuperscript{b} School of Distance Education, Universiti Sains Malaysia, 11800 Penang, Malaysia

\textsuperscript{*}Corresponding author, e-mail: missheikh92@gmail.com

Received 7 Jan 2013
Accepted 5 Apr 2013

ABSTRACT: In this paper, a class of quasi-quintic trigonometric Bézier curve with two shape parameters, based on newly constructed trigonometric basis functions, is presented. The new basis functions share the properties with Bernstein basis functions, so the generated curves inherit many properties of traditional Bézier curves. The presence of shape parameters provides a local control on the shape of the curve which enables the designer to control the curve more than the ordinary Bézier curve.

KEYWORDS: trigonometric basis functions, shape control, open curves, closed curves

INTRODUCTION

In computer aided geometric design (CAGD), it is often necessary to generate curves and surfaces that approximate shapes with some desired shape features. Designing free form curves and surfaces is a prevalent feature of CAGD. The key problem, simply stated, is to enable the designer to generate curves and surfaces which behave as he wants them to. The parametric cubic is a powerful tool which, when properly defined, is capable of representing most geometric entities of practical interest. In recent years, the trigonometric spline with shape parameters has gained more interest in CAGD, in particular curve design. Han\textsuperscript{1} presented a class of quadratic trigonometric polynomial curves with a shape parameter. The shape of the curve was more easily controlled by altering the values of shape parameter than the ordinary quadratic B-Spline curves. Han\textsuperscript{2} introduced piecewise quadratic trigonometric polynomial curves with \(C^1\) continuity analogous to the quadratic B-Spline curves which have \(C^1\) continuity. Cubic trigonometric polynomial curves with a shape parameter were discussed by Han\textsuperscript{3}. In these papers the authors described the trigonometric polynomial with global shape parameter. Single parameter does not provide local control on the curves. To remedy this, Wu et al\textsuperscript{4} presented quadratic trigonometric polynomial curves with multiple shape parameters.

Bézier technique is one of the methods of analytic representation of curves and surfaces that has won wide acceptance as a valuable tool in CAD/CAM system. They are used to produce curves which appear reasonably smooth at all scales. Today, many CAGD systems feature Bézier curves as their major building block, since they are very efficient and attain a number of mathematical properties. Both rational and non-rational forms of Bézier curves have been studied by many authors. A cubic trigonometric Bézier curve with two shape parameters was presented by Han et al\textsuperscript{5}. It enjoyed all the geometric properties of the ordinary cubic Bézier curve and was used for spur gear tooth design with S-shaped transition curve Abbas, et al\textsuperscript{6}. Liu, et al\textsuperscript{7} presented a study on class of TC-Bézier curve with shape parameters. Yang, et al\textsuperscript{8} discussed trigonometric extension of quartic Bézier curves attaining \(C^2\) and \(C^2\) continuity. A class of general quartic spline curves with shape parameters were introduced by Han\textsuperscript{9}. Yang, et al studied a class of quasi-quartic trigonometric Bézier curves and surfaces\textsuperscript{10}.

A newly constructed quasi-quintic trigonometric Bézier curve with two shape parameters is presented in this paper. The proposed curve inherits all geometric properties of the traditional Bézier curve and is used to construct open and closed curves.

The paper is organized as follows. Firstly, we construct new trigonometric Bernstein-like basis functions with two shape parameters. Secondly, a quasi-quintic trigonometric Bézier curve is constructed from these basis functions. Thirdly, we describe the local shape control of the curve using different values of shape parameters. This is then used to generate open and closed curves. Finally, the conclusion with future
Cubic Bezier Constrained Curve Interpolation

Fuziatul Norsyiha Ahmad Shukri\textsuperscript{a}, Muhammad Abbas\textsuperscript{b}, Uzma Bashir\textsuperscript{b}, Mohd Nain Hj Awang\textsuperscript{c}, Ena Jamal\textsuperscript{b}, Jamaludin Md Ali\textsuperscript{b}

\textsuperscript{a}School of Mathematical Sciences, Universiti Technology MARA, Macang, Kelantan, Malaysia
\textsuperscript{b}School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia
\textsuperscript{c}School of Distance Education, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia

ABSTRACT

Bézier function is one of the substantial polynomial and fundamental tool for interpolation because it is easy to compute and implement. In this paper, we develop cubic Bézier constrained curve interpolation. The end points of cubic Bézier function are left for user's choice. Simple constraints are derived on two middle points of cubic Bézier function constrained by a circle, an ellipse and straight line with point of intersection. Furthermore, the cubic Bézier function represents the S-shaped and C-shaped curves. The developed scheme is tested through different numerical examples and found to be computationally economical and visually pleasant.

KEY WORDS: Computer Aided Geometric Design, Cubic Bézier function, Interpolation, End points, S-shaped curve, C-shaped curve.

1. INTRODUCTION

Curves and surfaces design is an important topic of CAGD (Computer Aided Geometric Design) and computer graphics. CAGD is concerned with algorithms for the design of smooth curves and surfaces and has efficient mathematical representation. Bézier is one of the imperative polynomial and important tool for interpolation. The Bézier interpolating curve always lies within the convex hull and never wanders from the control polygon. Bézier polynomial has several applications in the fields of engineering, science and technology such as highway or railway rout designing, networks, Computer aided design system, animation, robotics, communications and many other disciplines because it is easy to compute and also stable [4,9,11]. The significance of Bézier polynomials in diverse areas, namely electronics or engineering is well known [6]. The parametric and non parametric representation of curves and surfaces especially in polynomial form is most suitable for design, as the planer curves cannot deal with infinite slopes and are axis dependent too.

Many authors have studied numerous kinds of Spline for curve and surface design, shape preservation [2, 7]. Abbas, et al [1], developed quadratic and cubic Bézier interpolations constrained by a line. The author derived simple conditions on the middle points of quadratic and cubic Bézier function to be constrained by a line. Abbas [2], developed a $C^1$ piecewise rational cubic function with shape parameters to preserve the shape of constrained data. Simple data dependent conditions on shape parameter were derived to preserve the shape of data lying above the straight line. Brodlie, et al [3] constructed modified quadratic Shepard method which interpolates a scattered data of any dimension to preserve the positivity. The authors inserted extra knots in the interval in such a way that the desired shape of data was preserved. Meek, et al [10], constructed a rational cubic for interpolating the given set of ordered points lying on one side of a polyline. Goodman, et al [5], developed two schemes of interpolating data to preserve the shape of data lying on one side of the straight line by non parametric rational cubic function. Firstly, they preserved the shape of data lying above the straight line by scaling the weights by some scale factor. Secondly, they preserved the shape of data by inserting new interpolation point.

Jeok, Ong [8] investigated $C^1$ monotonicity and $G^1$ constrained to lie on the same side of given constraint line using cubic Bézier-like function. Hussain, et al [7], developed $C^1$ piecewise rational cubic function in most general form to visualize the constrained data in the view of constrained curve that is lying above the straight line.

In this paper, we construct ordinary cubic Bézier function which is constrained by general circle, general ellipse, straight line and circle, straight line and an ellipse with point of intersection. Simple conditions are imposed on the two middle points of cubic Bézier function to be constrained by a straight line, circle and an ellipse with point of intersection. The cubic Bézier function has many advantages as compared to rational cubic function with shape parameter. Due to non rational form of the function, it is easy to compute and implement for two dimensional data. There is no need to insert additional knots where the function loses its shape. Likewise no constraint interval length is required as in rational cubic interpolations. The developed method is computationally economical, time saving and produces pleasing graphical results.

*Corresponding author: Muhammad Abbas, Department of Mathematics, University of Sargodha, Sargodha-Pakistan, Tel.: +60-12-4047316; Fax: +60-4-6532068. E-mail address: m.abbas@uos.edu.pk
Local Convexity Shape-Preserving Surface Data Visualization by Spline Function

Muhammad Abbas1*, Ahmad Abd Majid2, Mohd Nain Hj Awang3 and Jamaludin Md Ali4

1,2,4 School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia.
3 School of Distance Education, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia.

Research Article

Abstract

The main purpose of this paper is the visualization of convex surface data to present a smooth, visually pleasing and interactive convexity preserving surfaces. The rational cubic function with three free parameters is extended to rational bi-cubic partially blended function to preserve the shape of convex surface data. The function involves twelve free parameters in each rectangular patch. Data dependent constraints are derived for four of these parameters to preserve the shape of convex surface data while other eight are left free to user for the refinement of convexity preserving surface of data. Moreover, the scheme under discussion is $C^2$, flexible, simple, local and economical as compared to existing schemes.

2010 Mathematics Subject Classification: 65D05, 65D07, 65D017, 65D018, 65U05, 65U07

Keywords: Shape-preserving interpolation, rational bi-cubic partially blended function, convex surface, convex surface data, free parameters.

1 Introduction

The study of curves and surfaces is a key element in computer aided geometric design (CAGD) that has been around for quite some time. The methods of CAGD have arisen from the need of efficient computer representation of practical curves and surfaces used in engineering design. Spline interpolation is a powerful tool in Computer Graphics, CAGD and Engineering as well. Therefore, in these fields, it is often desirable to generate a convexity preserving interpolating curve and surface according to the given convex data. The aspiration of this paper is to preserve the hereditary attribute that is the convexity of data.

*Corresponding author: m.abbas@uos.edu.pk;
Positivity-preserving $C^2$ rational cubic spline interpolation

Muhammad Abbas$^{a,*}$, Ahmad Abd Majid$^a$, Mohd Nain Hj Awang$^b$, Jamaludin Md Ali$^a$

$^a$ School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang Malaysia
$^b$ School of Distance Education, Universiti Sains Malaysia, 11800 Penang Malaysia

*Corresponding author, e-mail: m.abbas@uos.edu.pk

Received 5 Jun 2012
Accepted 20 Mar 2013

ABSTRACT: This work addresses the shape preserving interpolation problem for visualization of positive data. A piecewise rational function in cubic/quadratic form involving three shape parameters is presented. Simple data dependent conditions for a single shape parameter are derived to preserve the inherited shape feature (positivity) of data. The remaining two shape parameters are left free for the designer to modify the shape of positive curves as per industrial needs. The interpolant is not only $C^2$, local, computationally economical, but it is also a visually pleasant and smooth in comparison with existing schemes. Several numerical examples are supplied to illustrate the proposed interpolant.

KEYWORDS: shape preserving interpolation, data visualization, positive data, shape parameters, parametric continuity

INTRODUCTION

The problem of constructing a shape preserving curve through given data points is one of the basic problems in computer graphics, computer aided geometric design, data visualization and engineering. Curve design plays a significant role not only in these fields but also in manufacturing different products such as ship design, car modelling, and aeroplane fuselages and wings.

A solution to the aforementioned problem results in the construction of some interpolants that preserve the inherited shape features of data such as positivity, monotonicity, and convexity. The goal of this paper is to preserve the genetic characteristic (positivity) of data. The positivity-preserving problem occurs in visualizing a physical quantity that cannot be negative which may arise if the data is taken from some scientific, social, or business environments.

Classical methods, with the polynomial spline functions, show smooth and visually pleasing results but usually ignore this shape feature of data and thus yield solutions exhibiting undesirable undulations or oscillations. Piecewise cubic Hermite interpolating polynomial (a built-in MATLAB program called PCHIP) has the ability to remove the undesired undulations but the shape preserving visual model depicts the tight display of data.

A great deal of research on this topic has been done, especially on the shape preserving interpolations. Some $C^1$ piecewise rational cubic interpolants$^1$ have a common feature in a way that no additional knots are used for shape preservation of positive 2D and 3D data. In contrast, the cubic Hermite interpolation$^6$ and cubic polynomial spline$^7$ preserve the shape of data by inserting one or two additional knots in the subinterval where the interpolants do not attain the desired shape characteristics of data. Some rational cubic interpolations$^8,9$ have been developed for value control, inflection-point control, convexity control of the interpolation at a point, and constrained curve control which were based on function values. The authors assumed suitable values of parameters to achieve a $C^2$ continuous curve and the methods work for only equally spaced data. Costantini$^{10}$ solved the shape preserving boundary valued problems using polynomial spline interpolation with arbitrary constraints.

The problem of shape preserving interpolants for visualization of positive, monotone and convex data has been solved using a $C^2$ rational cubic function with shape parameters$^{11,12}$. Simple data dependent constraints for free parameters were derived to achieve the desired shape features of the data. The schemes$^4,12$ did not provide the liberty to the designer for the refinement of shape of curves as per industrial needs. Lamberti and Manni$^{13}$ presented and investigated the approximation order of a global $C^2$ shape preserving interpolating function using parametric cubic curves. The tension parameters were used to control the shape of the curve. The authors derived the necessary and sufficient conditions for
Shape Preserving for 3D Positive Data by Spline Functions

Muhammad Abbas\textsuperscript{1}, Ahmad Abd Majid\textsuperscript{2}

Mohd Nain Hj Awang\textsuperscript{3} and Jamaludin Md Ali\textsuperscript{4}

\textsuperscript{124}School of Mathematical Sciences
\textsuperscript{3}School of Distance Education
University Sains Malaysia, 11800 USM, Penang, Malaysia
\textsuperscript{2}majid@cs.usm.my; \textsuperscript{3}mnain@usm.my; \textsuperscript{4}jamaluma@cs.usm.my

Abstract

In this paper, we made a visualization of positive data in such a fashion where it presents a smooth, pleasant and eye catching view of the positive surface to viewer. An attempt has been made in order to upgrade rational cubic function into a rational bi-cubic function for the preservation of positive data arranged over rectangular grid in the vision of positive surface. Moreover, rational bi-cubic function has six free parameters which are arranged in such a manner, two of them are constrained parameters to preserve the positive display of the positive data and remaining four are left for user choice to refine the positive surface as desired. The scheme under discussion is $C^1$, simple, local, computationally economical and time saving as compared to existing schemes. Numerical examples are provided to demonstrate that the anticipated scheme is interactive and smooth.

Mathematics Subject Classification: 68U05, 68U07, 65D05, 65D07, 65D18

Keywords: Visualization, Rational cubic function, Rational Bi-cubic function, Positive Surface, Positive data, Free parameters

1 Introduction

Spline interpolation is very powerful tool in Computer Graphics, Computer Aided Geometric Design and Engineering as well. Therefore, in these fields,
Designing Joint Font Using Cubic $\beta$-spline Curve With $G^1$ Continuity

Mohd. Nain Hj. Awang $^1$, Mustafa Mamat $^2$, Emi Marlina Mohd. Nain $^3$

$^1$Centre for Distance Education, Universiti Sains Malaysia, Penang.
$^2$Department of Mathematics, Universiti Malaysia Terengganu, Terengganu.
$^3$Mara Junior Science College, Perlis.

E-mail: $^1$mnain@usm.my, $^2$mus@umt.edu.my, $^3$emi_marlina84@yahoo.com.my.

Abstract

The aim of this paper is to discuss the designing of joint font using cubic $\beta$-spline curve with $G^1$ continuity. The aspect of geometric continuity is looked in this study due to the importance of shaping the desired curve. The step taken is to develop a $G^0$ continuity $\beta$-spline curve and then followed by a $G^1$ continuity $\beta$-spline curve. $\beta$-spline curve uses bias and tension parameters for controlling the shape of the desired curves. For this paper, the joint fonts are generated using $\beta$-spline curve and some comparison has been made between these curves in the context of smoothness and original shape. Moreover, the comparison between the joint font generated using $\beta$-spline curve and B-spline curve has also been considered.

Key words: Joint font, $\beta$-spline, $G^1$ continuity
Monotonicity Preserving Interpolation using Rational Spline

Muhammad Abbas, Ahmad. Abd Majid, Mohd Nain Hj Awang, Jamaludin Md. Ali

Abstract—The main spotlight of this work is to visualize the monotone data to envision of very smooth and pleasant monotonicity preserving curves by using piecewise rational cubic function. The piecewise rational cubic function has three shape parameters in each interval. We derive a simpler constraint on shape parameters which assure to preservation of the monotonicity curve of monotonic data. The free shape parameters will provide the extra freedom to the user to interactively generate visually more pleasant curves as desired. The smoothness of the interpolation is C1 continuity.

Index Terms—Rational cubic function, monotone data, shape parameters, free shape parameters, Interpolation.

I. INTRODUCTION

Smoothness and pleasant shape preserving curves are very important in Computer aided Geometric design (CAGD), Computer Graphics (CG), and Data Visualization (DV). In these fields, it is often needed to produce a monotonicity preserving interpolating curve corresponding to the given monotone data. The one very important feature from the interpolation methods is that make good judgment to study its positivity and monotonicity which are important aspects of the shape. Many physical situations where entities are taken only monotone. In [14], monotonicity is applied in the specification of Digital to Analog Converters (DACs), Analog to Digital Converters (ADCs) and sensors. These devices are used to control system applications where non monotonicity is not acceptable. In cancer patients, Erythrocyte sedimentation rates (ESR) and Uric acid level in blood are providing monotone data see[14]. Approximation of couples and quasi couples in statistics see [16], rate of dissemination of drug in blood see [16], empirical option of pricing models in finance see [16] are good examples of monotone data. Also in Engineering, the study of tensile strength of the material which give the monotone data because the tensile strength of a material can be defined as the maximum force that a material can withstand before breaking see for detail [12]. The forced applied usually is called stress and is studied alongside the stretch of the material referred as strain. So the data from these two entities is always monotone see for detail [12].

The problem of monotonicity preserving has been considered by number of authors [1]-[19] and references therein. Hussain & Sarfraz [1] has developed monotonicity preserving interpolation by rational cubic function with four shape parameters which are arranged as two are free and the other two are automated shape parameters. The authors derived data dependent constraints on automated shape parameters which ensure the monotonicity and also provide very pleasant curves but one automated shape parameter is dependent on the other and hence the scheme is economically very expensive.

Fritich & Carlson [2] and Butland [3] have developed a piecewise cubic polynomials monotonicity preserving interpolation. With positive derivatives and increasing data in the cubic Hermite Polynomials they still violate the necessary monotonicity condition. These authors are made a modification in input derivatives in their interpolating schemes if they contravene the necessary monotonicity conditions. Schumaker [5] developed a very economical interpolation by piecewise quadratic polynomial but the author also for interpolation they put in an extra knot in each interval. The constructed interpolant preserve the monotonicity but have degree 2. Butt [18] & Higham [4] also used a piecewise cubic polynomials in which the derivatives are not modified but the authors inserted extra knots where the curve lost the monotonicity. Gregory & Delbourgo [6] has introduced a solution of monotonicity without inserting an extra knot and preserves monotonicity by rational quadratic function with quadratic denominator. Fahr and Kallay [7] used a monotone rational B-spline of degree one to preserve the shape of monotone data.

For the pithiness of monotonicity preserving interpolation the reader is referred to: Delbourgo & Gregory [8] has developed piecewise rational cubic interpolation for the preserving monotonicity with no freedom to user to refine the curve. Gregory & Sarfraz [9] introduced a rational cubic spline with one tension parameter in each subinterval. Sarfraz [10] & [11], Hussain & Hussain [12] and Sarfraz & Hussain [13], Sarfraz et al [17] have introduced rational cubic function with two shape parameters that provide the desired monotone curves but there is no freedom for the user to modify the curves hence is may be unsuitable for interactive design. Hussain et al [14] also developed a rational function for the preserving monotonicity with single shape parameter which is very economical and generates pleasant curve but have limited flexibility in particular curves modification. Sarfraz [15] introduced an C2 interpolant using a general piecewise rational cubic (GPRC) with two shape parameters to preserves the monotonicity but there is also no freedom to the user.

Manuscript received September 03, 2010; revised December 31, 2010. This work was fully supported by the Universiti Sains Malaysia under Grant FRGS/203/JAUIH/6711120.

Muhammad Abbas is with the School of Mathematical Sciences, Universiti Sains Malaysia, USM 11800 Pulau Pinang, Malaysia, Corresponding E-mail: m.abbas@uus.edu.my.

Ahmad. Abd Majid is with the School of Mathematical Sciences, Universiti Sains Malaysia, USM 11800 Pulau Pinang, Malaysia.

Mohd Nain Hj Awang is with the School of Distance Education, Universiti Sains Malaysia, USM 11800 Pulau Pinang, Malaysia.

Jamaludin Md. Ali is with the School of Mathematical Sciences, Universiti Sains Malaysia, USM 11800 Pulau Pinang, Malaysia.
Subdivision Algorithm For The Triangular Spline Surface

Mohd. Nain Hj. Awang¹, Emi Marlina Mohd. Nain²

¹Centre for Distance Education, Universiti Sains Malaysia, 11800, Penang, Malaysia. ²Department of Mathematics, Universiti Malaysia Terengganu, 21030, Terengganu, Malaysia.

E-mail: ¹mnain@usm.my, ²emi_marlina8@ yahoo.com.my.

Abstract

Subdivision algorithms for rendering of box spline surfaces have been independently developed by Boehm, Cohen, Lyche, Riesenfeld, Dahmen, Michelli and Prautzsch. The algorithm refine the control net of any box spline surface so that the refined control nets converge to the spline. The aim of this paper is to study multivariate B-splines and triangular spline surface. The study will consider the divided difference of a function f which can be expressed in terms of a multiple integral. From here, we give a geometric interpretation of B-splines and the definition of triangular spline on uniform mesh. Subdivision algorithms for rendering of triangular spline surfaces are developed and the triangular spline surfaces are then generated.

Key words: Subdivision algorithm, triangular spline, uniform mesh.

1. 1 Introduction

Triangular polynomial patches were first considered by de Casteljau in Computer Aided Geometric Design (CAGD) (Farin, 1983), but these scarcely received any attention. The triangular patches were generated based on the Bezier polynomials defined over the arbitrary triangles. Sabin used triangular patches in Bernstein form to construct B-splines over regular triangular by convolution (Sabin, 1977). Later, the B-splines were found to be the triangular spline (de Boor and de Vore, 1983). Subdivision algorithms for rendering of triangular spline surfaces have been independently developed by Boehm, Cohen, Lyche, Riesenfeld, Dahmen, Michelli and Prautzsch (Cohen, Lyche and Riesenfeld, 1984).

1. 2 Triangular Splines On A 3-Direction Mesh

Let

\[ V = \{ \varepsilon^1, \varepsilon^2, \ldots, \varepsilon^k \} \subseteq \mathbb{R}^2, \ k \geq 2, \]

where \( \varepsilon^1 = (1, 0), \varepsilon^2 = (0, 1), \varepsilon^3, \ldots, \varepsilon^k \in \{ \varepsilon^1, \varepsilon^2, \varepsilon^2 - \varepsilon^1 \}, \) and suppose span \( [\varepsilon^1, \varepsilon^2] = \mathbb{R}^2. \)